Chapter 16—One-way Analysis of Variance

I am assuming that most people would prefer to see the solutions to these problems as computer printout. (I will use R and SPSS for consistency.)

16.1 Analysis of Eysenck's data:

a) The analysis of variance:

```
               - - - - - O N E W A Y - - - - -
  Variable  RECALL
  By Variable  GROUP     Group Membership
                      Analysis of Variance
                              Sum of       Mean          F        F
      Source          D.F.     Squares     Squares      Ratio    Prob.
Between Groups          1     266.4500    266.4500    25.2294  .0001
Within Groups          18     190.1000     10.5611
Total                  19     456.5500

                           Standard   Standard
Group      Count     Mean  Deviation    Error    95 Pct Conf Int for Mean

Grp 1       10      19.3000    2.6687     .8439    17.3909  TO  21.2091
Grp 2       10      12.0000    3.7417    1.1832     9.3234  TO  14.6766

Total      20       15.6500    4.9019    1.0961    13.3558  TO  17.9442
```

b) *t* test

```
t-tests for Independent Samples of GROUP    Group Membership
                         Number
 Variable              of Cases       Mean          SD   SE of Mean
 ----------------------------------------------------------------------
 RECALL
 Young                     10        19.3000       2.669     .844
 Older                     10        12.0000       3.742    1.183
 ----------------------------------------------------------------------
          Mean Difference = 7.3000
         Levene's Test for Equality of Variances: F= .383   P= .544

       t-test for Equality of Means                            95%
 Variances   t-value    df    2-Tail Sig     SE of Diff    CI for Diff
 ----------------------------------------------------------------------
 Equal       5.02       18        .000        1.453    (4.247, 10.353)
 Unequal     5.02       16.27     .000        1.453    (4.223, 10.377)
```

Notice that if you square the *t* value of 5.02 you obtain 25.20, which is the same as the *F* in the analysis of variance. Notice also that the analysis of variance procedure produces confidence limits on the means, whereas the *t* procedure produces confidence limits on the difference of means.

16.3 Expanding on Exercise 16.2:

a) Combine the Low groups together and the High groups together:

```
      Variable  RECALL
   By Variable  LOWHIGH
                                  Analysis of Variance
                              Sum of        Mean          F      F
        Source         D.F.   Squares       Squares       Ratio  Prob.
Between Groups           1    792.1000     792.1000      59.4505  .0000
Within Groups           38    506.3000      13.3237
Total                   39   1298.4000

                        Standard   Standard
Group   Count    Mean   Deviation     Error    95 Pct Conf Int for
Mean

Grp 1    20    6.7500    1.6182     .3618     5.9927  TO    7.5073
Grp 2    20   15.6500    4.9019    1.0961    13.3558  TO   17.9442

Total    40   11.2000    5.7699     .9123     9.3547  TO   13.0453
```

Here we have compared recall under conditions of Low versus High processing, and can conclude that higher levels of processing lead to significantly better recall.

b) The answer is still a bit difficult to interpret because both groups contain both younger and older subjects, and it is possible that the effect holds for one age group but not for the other.

d) When we assume equal variances $t^2 = 4.34^2 = 18.84$. When we assume unequal variances $t^2 = 4.27^2 = 18.23$. Within rounding error the *F* corresponding to the *t* with pooled variances (the *t* assuming equal variances) is equal to the *F* from the analysis of variance.

You could point out to students that the analysis of variance always uses the equivalent of a pooled variance term unless you go in with your calculator and deliberately calculate it in some other way.

16.5 η^2 and ω^2 for the data in Exercise 16.1:

$SS_{group} = 266.45$
$SS_{total} = 456.55$
$MS_{error} = 10.564$
$k = 2$

$$\eta^2 = \frac{SS_{group}}{SS_{total}} = \frac{266.45}{456.55} = .58$$

$$\omega^2 = \frac{SS_{group} - (k-1)MS_{error}}{SS_{total} + MS_{error}}$$

$$= \frac{266.45 - (2-1)10.564}{456.55 + 10.564} = \frac{255.886}{467.114} = .55$$

Here is another illustration that η^2 and ω^2 are often quite close. You could start a discussion from the fact that there are several exercises that require students to compute magnitude of effect measures, and those measures vary substantially from one experiment to another. This could lead to a discussion of when a measure, such as η^2, is too low to be meaningful or two high to be anything but trivial.

16.7 Foa *et al*. (1991) study:

Group	***n***	**Mean**	**S.D.**	**Total**	**Variance**
SIT	14	11.07	3.95	155	15.6025
PE	10	15.40	11.12	154	123.6544
SC	11	18.09	7.13	199	50.8369
WL	10	19.50	7.11	195	50.5521
Total	45	15.622		703	

$$\bar{X}_{..} = \frac{703}{45} = 15.622$$

$$SS_{treat} = \sum n_j \left(\overline{X}_j - \overline{X}..\right)^2$$

$$= 14(11.07 - 15.622)^2 + 10(15.40 - 15.622)^2 + 11(18.09 - 15.622)^2 + 10(19.50 - 15.622)^2$$

$$= 507.840$$

$$MS_{error} = \frac{\sum (n_i - 1)s_i^2}{\sum (n_i - 1)}$$

$$= \frac{13(15.6025) + 9(123.6544) + 10(50.8369) + 9(50.5521)}{41}$$

$$= 55.587$$

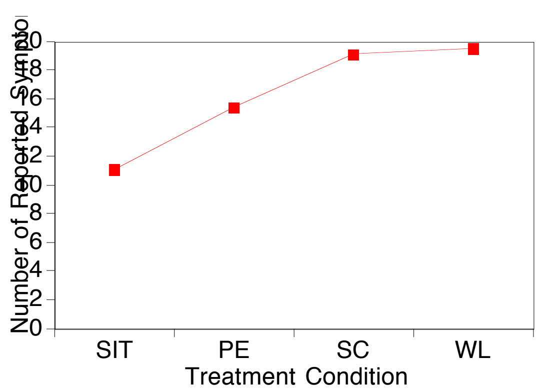
$$SS_{error} = [\Sigma(n_1 - 1)]MS_{error} = 41 * 55.587 = 2279.067$$

From these values we can fill in the complete summary table and compute the *F* value.

Source	*df*	*SS*	*MS*	*F*
Treatment	3	507.840	169.280	3.04
Error	41	2279.067	55.587	
Total	44	2786.907		

[$F_{.05}(3,41) = 2.84$] We can reject the null hypothesis and conclude that there are significant differences between groups. Some treatments are more effective than others.

b)



c) It would appear that the more interventionist treatments lead to fewer symptoms than the less interventionist ones, although we would have to run multiple comparisons to tell exactly which groups are different from which other groups.

> You might remind students that these are the results of an actual experiment. Some forms of therapy are better than others, and are better than a no-treatment control. We sometimes lose sight of that.

16.9 *R* code for Ex16.7

This code generates random data, so the means and standard deviations will not be exact. But the set.seed(3086) should produce a result that is significant.

```
# Exercise 16.9
# Generate data
set.seed(3086)
ST <- round(rnorm(14, 11.07, 3.95), digits = 2)
PE <- round(rnorm(10, 15.40, 11.12), digits = 2)
SC <- round(rnorm(11, 18.09, 7.13), digits = 2)
WL <- round(rnorm(10, 19.5, 7.11), digits = 2)
dv <- c(ST, PE, SC, WL)
group <- factor(a <- rep(c(1,2,3,4), c(14, 10, 11, 10)))
model <- lm(dv ~ group)
anova(model)
```

16.11 If the sample sizes in Exercise 16.7 were twice as large, that would double the SS_{treat} and MS_{treat}. However it would have no effect on MS_{error}, which is simply the average of the group variances. The result would be that the *F* value would be doubled.

16.13 *R* code for analysis of Exercise 16.2

```
#Ex16.13
data <- read.table("https://www.uvm.edu/~dhowell/fundamentals9/DataFiles/
  Tab16-1.dat", header = TRUE)
attach(data)
group <- factor(group)         # IMPORTANT! Specify that group is a factor
model1 <- lm(dv ~ group)    # Calculate the linear model of dv predicted from
group
anova(model1)
16.13  Effect size for tests in Exercise 16.10.
```

16.15 It only makes sense to calculate an effect size for significant comparisons in this study, so we will deal with SIT vs SC.

$$\hat{d} = \frac{\bar{X}_{SC} - \bar{X}_{SIT}}{\sqrt{MS_{error}}} = \frac{18.09 - 11.07}{\sqrt{55.579}} = \frac{7.02}{7.455} = 0.94$$

The SIT group is nearly a full standard deviation lower in symptoms when compared to the SC group, which is a control group.

16.17 ANOVA on GPAs for the ADDSC data:

Variable GPA
By Variable Group

Source	D.F.	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between Groups	2	22.5004	11.2502	22.7362	.0000
Within Groups	85	42.0591	.4948		
Total	87	64.5595			

Group	Count	Mean	Standard Deviation	Standard Error	95 Pct Conf Int for Mean
Grp 1	14	3.2536	.5209	.1392	2.9528 TO 3.5543
Grp 2	49	2.5920	.6936	.0991	2.3928 TO 2.7913
Grp 3	25	1.7436	.8020	.1604	1.4125 TO 2.0747
Total	88	2.4563	.8614	.0918	2.2737 TO 2.6388

Using *R*

```
Analysis of Variance Table

Response: GPA
          Df Sum Sq Mean Sq F value    Pr(>F)
grp        2 22.500 11.2502  22.736 1.232e-08 ***
Residuals 85 42.059  0.4948
---
```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

There is a significant difference between the groups, telling us that there is a relationship between ADDSC score in elementary school and the GPA the student has in 9th grade. From the means it is clear that the GPA declines as the ADDSC score increases.

> These are real data, and they tell us that a teacher in elementary school can already pick out those students who will do well and badly in high school. I have always found these results depressing and worrisome, even though psychologists are supposed to like to be able to predict. There are some things I wish weren't so predictable.

16.19 Analysis of Darley and Latané data:

Group	***n***	**Mean**	**Total**
1	13	0.87	11.31
2	26	0.72	18.72
3	13	0.51	6.63
Total	52		36.66

$$SS_{treat} = \Sigma n_j\left(\bar{X}_j - \bar{X}..\right)^2$$
$$= 13\left(0.87 - 0.705\right)^2 + 26\left(0.72 - 0.705\right)^2 + 13\left(0.51 - 0.705\right)^2$$
$$= 0.8541$$
$$MS_{error} = 0.053 \quad \text{(given in text)}$$
$$SS_{error} = [\Sigma(n_1 - 1)]MS_{error} = 49 * 0.053 = 2.597$$

From these values we can fill in the complete summary table and compute the F value.

Source	***df***	***SS***	***MS***	***F***
Treatment	2	0.854	0.427	8.06
Error	49	2.597	0.053	
Total	51	3.451		

[$F_{.05}(2,49) = 3.18$] We can reject the null hypothesis and conclude that subjects are less likely to summon help quickly if there are other bystanders around.

16.21 Bonferroni test on data in Exercise 16.2:

Both of these comparisons will be made using t tests. The means are given in Exercise 16.15 above.

$$t = \frac{\overline{X}_i - \overline{X}_j}{\sqrt{\frac{MS_{error}}{n_i} + \frac{MS_{error}}{n_j}}}$$

For Young/Low versus Old/Low:

$$t = \frac{6.5 - 7.0}{\sqrt{\frac{6.6278}{10} + \frac{6.6278}{10}}} = \frac{-0.5}{1.151} = -0.434$$

For Young/High versus Old/High:

$$t = \frac{19.3 - 12.0}{\sqrt{\frac{6.6278}{10} + \frac{6.6278}{10}}} = \frac{7.3}{1.151} = 6.34$$

For 36 *df* for error and for 2 comparisons at a familywise error rate of $\alpha = .05$, the critical value of $t = 2.34$. There is clearly not a significant difference between young and old subjects on tasks requiring little cognitive processing, but there is a significant difference for tasks requiring substantial cognitive processing. The probability that *at least* one of these statements represents a Type I error is at most .05.

> It is worth pointing out to students that when we are using MS_{error} as our variance estimate, and have equal sample sizes, the computations are very simple because we only need to calculate the denominator once.

16.23 Effect size for WL versus SIT

$$\hat{d} = \frac{\overline{X}_{WL} - \overline{X}_{SIT}}{s_{WL}} = \frac{19.50 - 11.07}{7.11} = \frac{8.43}{7.11} = 1.18$$

The two groups differ by over a standard deviation.

16.25 Spilich *et al*. data on a cognitive task:

```
Variable  ERRORS
   By Variable  SMOKEGRP
                                  Analysis of Variance

                             Sum of         Mean             F      F
      Source        D.F.    Squares        Squares         Ratio  Prob.
Between Groups        2     2643.3778     1321.6889       4.7444  .0139
Within Groups        42    11700.4000      278.5810
Total                44    14343.7778

                        Standard   Standard
Group    Count   Mean   Deviation    Error    95 Pct Conf Int for Mean

Grp 1      15   28.8667    14.6866     3.7921     20.7335  TO  36.9998
Grp 2      15   39.9333    20.1334     5.1984     28.7838  TO  51.0828
Grp 3      15   47.5333    14.6525     3.7833     39.4191  TO  55.6476

Total      45   38.7778    18.0553     2.6915     33.3534  TO  44.2022
```

Here we have a task that involves more cognitive involvement, and it does show a difference due to smoking condition. The non-smokers performed with fewer errors than the other two groups, although we will need to wait until the next exercise to see the multiple comparisons.

16.27 Spilich *et al*. data on driving simulation:

```
Variable  ERRORS
   By Variable  SMOKEGRP
                                 Analysis of Variance

                         Sum of         Mean             F      F
    Source        D.F.    Squares        Squares         Ratio  Prob.
Between Groups      2      437.6444      218.8222       9.2584  .0005
Within Groups      42      992.6667       23.6349
Total              44     1430.3111

                        Standard   Standard
Group    Count     Mean  Deviation  Error      95 Pct Conf Int for Mean

Grp 1      15    2.3333    2.2887      .5909      1.0659  TO       3.6008
Grp 2      15    6.8000    5.4406     1.4048      3.7871  TO       9.8129
Grp 3      15    9.9333    6.0056     1.5506      6.6076  TO      13.2591

Total      45    6.3556    5.7015      .8499      4.6426  TO       8.0685
```

Here we have a case in which the active smokers again performed worse than the non-smokers, and the differences are significant.

16.29 Attractiveness of faces

a) The research hypothesis would be the hypothesis that faces averaged over more photographs would be judged more attractive than faces averaged over fewer photographs.

b) Data analysis

Descriptives

ATTRACT

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean Lower Bound	95% Confidence Interval for Mean Upper Bound	Minimum	Maximum
1.00	6	2.60467	.431353	.176099	2.15199	3.05734	2.201	3.380
2.00	6	2.64500	.657059	.268243	1.95546	3.33454	1.893	3.644
3.00	6	2.89000	.447100	.182528	2.42080	3.35920	2.118	3.422
4.00	6	3.18500	.208053	.084937	2.96666	3.40334	2.860	3.505
5.00	6	3.26000	.068118	.027809	3.18852	3.33148	3.169	3.357
Total	30	2.91693	.473378	.086427	2.74017	3.09370	1.893	3.644

ANOVA

ATTRACT

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	2.170	4	.543	3.134	.032
Within Groups	4.328	25	.173		
Total	6.499	29			

c) Conclusions

The group means are significantly different. From the descriptive statistics we can see that the means consistently rise as we increase the number of faces over which the composite was created.

16.31 Analysis EX.27 using *R*

```
data16.27 <-
read.table("http://www.uvm.edu/~dhowell/fundamentals9/DataFiles/Ex16-
25.dat", header = TRUE)
attach(data16.27)
Smkgrp <- factor(Smkgrp)
```

```
model2 <- lm(Errors ~ Smkgrp)
anova(model2)
```

```
Analysis of Variance Table
Response: Errors
           Df  Sum Sq  Mean Sq  F value   Pr(>F)
Smkgrp      2  437.64  218.822   9.2584   0.0004665 ***
Residuals  42  992.67   23.635
```

16.32 Probability value for Ex16.31

```
prob <- 1-pf(9.258, df1 = 2, df2 = 42)
prob
[1] 0.000466617
```